九十五學年第二學期 PHYS2320 電磁學 期中考 I 試題(共兩頁)

[Griffiths Ch. 7-8] 2007/04/10, 10:10am–12:00am, 教師：張存績

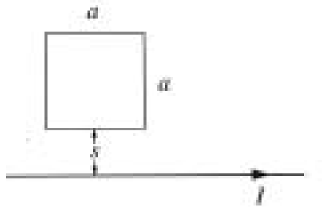
記得寫上學號，班別及姓名等。請依題號順序每頁答一題。

1. (6%, 7%, 7%) A square loop of wire (side a) lies on a table, a distance s from a very long straight wire, which carries a current I, as shown in the figure.

   (a) Find the flux of **B** through the loop.

   (b) If the loop is pulled away from the wire at speed v, what emf is generated? In what direction does the current flow?

   (c) If the current is slowly varying in time I(t), determine the induced electric field (amplitude and direction), as a function of the distance s from the wire.



2. (7%, 7%, 6%) A toroidal coil with rectangular cross section (inner radius a, outer radius b, height h, and a total of N turns) carries a current I.

   (a) Calculate the magnetic energy stored in the toroidal coil.

   (b) Calculate the flux through a single turn.

   (c) Find the self-inductance L of the toroidal coil.

3. (10%, 10%)

   (a) Write down Maxwell's equations in matter in terms of free charges ρ_f and current $\mathbf{J}_f$. Also for linear media, write the appropriate constitutive relations, giving **D** and **H** in terms of **E** and **B**.

   (b) Write down the four boundary conditions ($E^{\perp}$, $E^{/\!/}$, $B^{\perp}$, and $B^{/\!/}$) for linear media, if there is no free charge and no free current at the interface.

4. (12%, 8%)

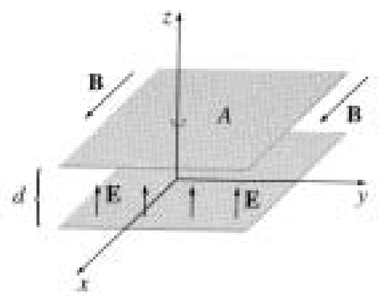
(a) Derive Poynting's theorem (the "work-energy theorem" of electrodynamics.) [Hint: $\frac{dW}{dt} = \int_V (\mathbf{E}\cdot\mathbf{J}_f) d\tau$ and $\mathbf{E}\cdot(\nabla\times\mathbf{H}) = -\nabla\cdot(\mathbf{E}\times\mathbf{H}) + \mathbf{H}\cdot(\nabla\times\mathbf{E})$]

(b) Write down the differential version of the Poynting theorem (conservation of energy). Explain the symbols you use as clear as possible.

5. (10%, 10%) A charged parallel-plates capacitor (with uniform electric field $\mathbf{E} = E_0\hat{\mathbf{z}}$) is placed in a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{x}}$, as shown in the figure.

(a) Find the amplitude and direction of the Poynting vector in the space between the plates.

(b) Suppose we slowly reduce the magnetic field. This will induce a Faraday electric field, which in turn exerts a force on the plates. Show that the total impulse is equal to the momentum originally stored in the field.



1. According to Ampere's law, the magnetic field is $\mathbf{B}=\frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\varphi}}$

(a) The total flux $\Phi=\int \mathbf{B}\cdot d\mathbf{a}=\frac{\mu_0 Ia}{2\pi}\int_s^{s+a}\frac{1}{s}ds=\frac{\mu_0 Ia}{2\pi}\ln(\frac{s+a}{s})$

(b) The flux change rate is equal to the total emf $\mathcal{E}=-\frac{d\Phi}{dt}=-\frac{\mu_0 Ia}{2\pi}(\frac{1}{s+a}-\frac{1}{s})\frac{ds}{dt}=\frac{\mu_0 Ia}{2\pi}\frac{av}{s(s+a)}$

According to Lenz's law, the current flows counterclockwise to compensate the flux loss.

(c)

$$\oint \mathbf{E}\cdot d\mathbf{l}=E(s_0)l-E(s)l=-\frac{d\Phi}{dt}=-\frac{\mu_0 l}{2\pi}\frac{dI(t)}{dt}(\ln s-\ln s_0)$$

$$E(s)=\frac{\mu_0}{2\pi}\frac{dI(t)}{dt}(\ln s-\ln s_0)+E(s_0)$$

$$\mathbf{E}(s)=(\frac{\mu_0}{2\pi}\frac{dI}{dt}\ln s+K)\hat{\mathbf{z}},\quad \text{where } K \text{ is a constant.}$$

See Example 7.9 on P.308.

2.

(a)

$$B=\frac{\mu_0 NI}{2\pi r}$$

$$dU_B=\frac{B^2}{2\mu_0}dV=\frac{B^2}{2\mu_0}h(2\pi r dr)=\frac{\mu_0 h(NI)^2}{4\pi r}dr$$

$$U_B=\int_a^b\frac{\mu_0 h(NI)^2}{4\pi r}dr=\frac{\mu_0 hN^2I^2}{4\pi}\ln(\frac{b}{a})$$

(b) The magnetic field inside a toroid $B=\frac{\mu_0 NI}{2\pi s}$

$$\Phi_1=h\int_a^b\frac{\mu_0 NI_1}{2\pi s}ds=\frac{\mu_0 hNI_1}{2\pi}\ln(b/a)$$

(c)

Use the flux relation: $L=\frac{N\Phi_1}{I_1}\ \Rightarrow\ L=\frac{\mu_0 hN^2}{2\pi}\ln(\frac{b}{a})$

Use the energy relation:

$$U_B=\int_a^b\frac{\mu_0 h(NI)^2}{4\pi r}dr=\frac{\mu_0 hN^2I^2}{4\pi}\ln(\frac{b}{a})=\frac{1}{2}LI^2$$

$$L=\frac{\mu_0 N^2 h}{2\pi}\ln(\frac{b}{a})$$

3.

(a)

$$\nabla\cdot\mathbf{D}=\rho_f \qquad \nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$$

$$\nabla\cdot\mathbf{B}=0 \qquad \nabla\times\mathbf{H}-\frac{\partial\mathbf{D}}{\partial t}=\mathbf{J}_f ,$$

where $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 (1+\chi_e)\mathbf{E} = \varepsilon \mathbf{E}$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \quad \Rightarrow \quad \mathbf{B} = \mu_0 (1+\chi_{\mathrm{m}})\mathbf{H} = \mu \mathbf{H}$$

(b) For linear media, $\mathbf{D} = \varepsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$

$$\begin{matrix} D_1^{\perp} - D_2^{\perp} = 0 & \mathbf{E}_1^{//} - \mathbf{E}_2^{//} = 0 \\ B_1^{\perp} - B_2^{\perp} = 0 & \mathbf{H}_1^{//} - \mathbf{H}_2^{//} = 0 \end{matrix} \quad \Rightarrow \quad \begin{matrix} \varepsilon_1 E_1^{\perp} - \varepsilon_2 E_2^{\perp} = 0 & \mathbf{E}_1^{//} - \mathbf{E}_2^{//} = 0 \\ B_1^{\perp} - B_2^{\perp} = 0 & \frac{1}{\mu_1}\mathbf{B}_1^{//} - \frac{1}{\mu_2}\mathbf{B}_2^{//} = 0 \end{matrix}$$

4. (a)

$$\frac{dW}{dt} = \int_V (\mathbf{E}\cdot\mathbf{J}_f) d\tau = \int_V (\mathbf{E}\cdot(\nabla\times\mathbf{H}) - \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t}) d\tau$$

Product rule: $\mathbf{E}\cdot(\nabla\times\mathbf{H}) = -\nabla\cdot(\mathbf{E}\times\mathbf{H}) + \mathbf{H}\cdot(\nabla\times\mathbf{E})$

Faraday's law: $\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

$$\begin{aligned} \frac{dW}{dt} &= \int_V [-\nabla\cdot(\mathbf{E}\times\mathbf{H}) - (\mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t} + \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t})] d\tau \\ &= -\int_S (\mathbf{E}\times\mathbf{H})\cdot d\mathbf{a} - \int_V (\mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t} + \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t}) d\tau \end{aligned} \quad \Rightarrow \quad S = \mathbf{E}\times\mathbf{H}$$

(b)

$$\frac{dW}{dt} = \frac{d}{dt}\int_V u_{\mathrm{mech}} d\tau \quad \text{and} \quad \frac{\partial u_{\mathrm{em}}}{\partial \mathrm{t}} = \mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t} + \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t} = \frac{\partial}{\partial t}(\frac{1}{2\mu}B^2 + \frac{1}{2}\varepsilon_0 E^2) \quad \text{for linear media}$$

$$\frac{\partial}{\partial t}(u_{\mathrm{mech}} + u_{\mathrm{em}}) = -\nabla\cdot\mathbf{S},$$

where u_{mech} is the mechanical energy density, u_{em} is the electromagnetic energy density, and $\mathbf{J}$ is the Poynting vector.

5.

(a) $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{1}{\mu_0} E_0 B_0 \hat{\mathbf{y}}$

(b)

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi}{dt} = -\frac{dB}{dt} ld$$

$$-l(E(d) - E(0)) = -\frac{dB}{dt} ld \quad \Rightarrow \quad E(d) - E(0) = d\,\frac{dB}{dt}$$

$$\mathbf{F} = \ -\sigma A(E(d) - E(0))\hat{\mathbf{y}} = \sigma A d\,\frac{dB}{dt}\hat{\mathbf{y}}$$

$$\mathbf{I} = \int_0^{\infty} \mathbf{F} dt = -\sigma A d(B(t=\infty) - B(t=0))\hat{\mathbf{y}} = \sigma A d B_0 \hat{\mathbf{y}}$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \Rightarrow \quad \mathbf{I} = \sigma A d B_0 \hat{\mathbf{y}} = \varepsilon_0 E_0 B_0 A d\hat{\mathbf{y}} \quad \text{as before}$$